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Data Flow Analysis of Parallel Programs*

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Abstract

Data flow analysis is the prerequisite of performing optimizations such as *code motion of partial redundant expressions* on imperative sequential programs. To apply these transformations to parallel imperative programs, the notion of data flow must be extended to concurrent programs. The additional parallel source language features are: shared memory and nested parallel statements (`PAR`). The underlying interleaving semantics of the concurrently-executed processes result in the so-called *state space explosion* which on first appearance prevents the computation of the *meet over all path* solution needed for data flow analysis.

For the class of bit-vector data flow problems we can show that for the computation of the *meet over all path* solution, not all interleavings are needed. Based on that, we can give simple data flow equations representing the data flow effects of the `PAR` statement. The definition of a *parallel control flow graph* leads to an efficient extension of Killdal's algorithm to compute the data flow of a concurrent program. The time complexity is the same as for analyzing a "comparable" sequential program.

Keywords: Data flow analysis, parallel languages.

1 Introduction and Motivation

The key tool for attacking the *Grand Challenge Problems* is parallelism. Parallel hardware is becoming more available and cheaper, but to program these devices is still a difficult task. Hence high-level programming languages are needed which have enough expressive power to implement parallel algorithms. But as usual with high-level languages, when translating them to machine code, some inefficiencies are introduced by the compiler. Therefore compilers have to perform so-called optimizations which improve the program. We may distinguish broadly between two kinds:

- transformations performed on the input language level such as *mapping of SIMD programs to MIMD machines*, *removal of unnecessary synchronization and communication*, *clustering of processes*, *data placement* etc. and
- transformations performed on the machine language level, such as *common subexpression elimination*, *constant folding*, *dead code elimination*, *code motion* etc.

For the rest of this paper we have only the second kind of optimizing transformations in mind.

Optimizing a program requires analyzing it, and this is often done by solving data flow equations for the program. Traditional data flow analysis (DFA) methods [10] are designed for sequential programs. Hence they may fail when applied to the control flow of parallel programs as shown in [14]. We give another example showing the problems when "low level" optimizations such as instruction scheduling are performed on parallel programs.

1.1 The Problem

Data flow analysis is more or less the estimation of the effects caused by program statements. This estimation is based on two things: an abstraction of the information needed as prerequisite for the optimizing transformation, and the propagation of the information along the statements of the source program. The information is usually represented by the elements of a semi-lattice[1]. The effect of a single program statement is then a function over these semi-lattice values. One execution of a program (up to the point of consideration) represents an execution path. The propagation is modeled by applying these functions in the order given by the statements of such an execution path. Since we are looking for the "worst case information" (only this guarantees that the transformation is correct for all execution paths leading to this program point) we have to consider the *Meet Over all Paths* of these information. [11] formalized this idea and gave an efficient algorithm to compute the data flow information for all points of a program.

The data flow information of two statement sequences, without any branches, executed concurrently, is given by the *meet* of the information of all interleavings[2] of the two sequences. It is clear that this may lead to a "state space explosion" [5], which makes it, on a very first view, intractable to compute the data flow information implied by a `PAR` statement.

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[1]E.g. boolean values for "the information is present or not".

[2]The topological sorting of the simple statements from both sequences.

1.2 Contribution of this Work

Hence we have to ask:

- which data flow information is valid after the PAR statement, and
- which information is valid before each statement in a process body?

The main contributions of our paper are:

- The lattice-theoretic based data flow framework is extended to cope with parallel programs. The proposed extension is valid only for the large class of *one-bit* (also known as *bit-vector*) data flow problems. They are based on a two-element semi-lattice.
- Simple bit-vector data flow equations are derived representing the data flow effects of the PAR statement.
- The Parallel Control Flow Graph is defined and used as a base for an extension of the well known and efficient iterative data flow analysis algorithm.

Based on these results, data flow analysis of parallel programs is possible and efficient. Then traditional optimizations may be applied to these programs without any restriction.

1.3 Related Work

Current approaches in analyzing the data flow of parallel programs have either a restricted model of shared memory, or even disallow it.

[17] investigates the data flow of communicating processes, but these do not share memory: processes communicate solely through synchronous channels. [19] describes an efficient method of computing the *Static Single Assignment Form* for explicitly parallel programs with `wait` clauses. The parallel sections must be data-independent, except where explicit synchronization is used. The same is true for [20, 26] who introduce a *Parallel Control Flow Graph* and the *Parallel Precedence Graph* which may form the basis of concrete optimizing algorithms.

[6] presents data flow equations for parallel programs, both with and without synchronization. But this work is restricted to PCF FORTRAN programs, which means that access to shared variables is done only at synchronization points. For process start and process end a copy in/copy out semantics is assumed. An intuitive but not formal derivation of the data flow equations is given, which solve only the *reaching definition* problem.

[4] extends abstract interpretation to cope with communicating sequential processes. The problem there is that the resulting "state space" explodes. [3] applies abstract interpretation to analysis of parallel programs, but bases the semantics on a *labeled transition system*. [5] attacks this problem using the *stubborn set theory* [21] which decreases the state set using some heuristics. Hence the analysis is accurate for some examples, and less accurate for others.

[23] presents the basic idea of how the number of interleavings may be reduced; the parallel program is represented by its structure tree. [25] uses ideas of this report to prove the *Hierarchical Coincidence Theorem* which is based on a functional representation [18] of the problem. The full version of this paper is available as [24].

2 Machine Model

We assume that other phases of the compiler have done the more "high-level" transformations already, and hence our investigation is based on an imperative language with explicit control flow parallelism, dynamic process creation, and shared memory. As a computing model we assume a MIMD (multiple instruction, multiple data) system, where each process is executed on a separate logical processor.[3] Each processor runs independently of each other and has its own set of registers, which are invisible for other processors. All processors access a shared memory. The access to a single memory cell is atomic, i.e. at a given time only one process may read or write a given cell. We assume an interleaving semantics for the execution of the program with respect to the memory accesses.

3 The Sample Language

A simple imperative language will be used in this paper, which has loops, conditional statements, and a statement to execute other statements in parallel (explicit control flow parallelism). Replicators allow dynamic process creation, and processes share memory.

The PAR statement executes all processes specified by ProcessBody in parallel and independently. The process executing a PAR statement[4] is suspended until all child processes have terminated. A ProcessBody is a list of statements, which may be replicated. That is: `max(UpBound - LowBound + 1, 0)` processes are forked which all execute the statements following the replicator. Each replicated process gets its private copy of the replicator variable `var`, which has in each replicated process a unique value in the range `[LowBound .. UpBound]`. Replicated processes are also called *asynchronous for-all loops* in other languages. All variables can be accessed in each process. No automatic synchronization is done for the access.

```
Stmt         ::= PAR ProcessBody//"|" END.
ProcessBody  ::= [Replicator] Stmt//";".
Replicator   ::= "["var":"LowBound TO UpBound"]".
LowBound     ::= Expr.
UpBound      ::= Expr.
Expr         ::= usual expressions.
```

`X//Y` is a list of X's separated by a `Y`. `[X]` stands for an optional `X` part.

4 Lattice-Theoretic Background of Data Flow Analysis

This section gives the lattice-theoretic background of data flow analysis and follows [9]. It may be skipped by the reader familiar with the notation.

The source program under consideration is represented as a (sequential) control flow graph[5]:

Definition 1 *A* control flow graph *is a triple* $G = (N, E, n_0)$, *where* N *is a finite set of* nodes *(which contains a list of simple statements, such as assignments).* $E \subseteq N \times N$ *is a set of ordered* edges *between these nodes and* n_0 *the unique initial node.*

[3]A *processor* may be implemented via a time sharing system.

[4]PAR statements may be nested.

[5]Section 8 defines the parallel version of a control flow graph.

A path *from n_1 to n_k is a sequence of nodes $n_1, n_2, \ldots, n_k$, such that for $1 \leq i < k$ all edges $(n_i, n_{i+1}) \in E$. Such a path has* length *k.*

For a node n pred$[n]$(succ$[n]$) *is the set of* predecessors (successors) *defined as:* pred$[n] = \{n' : (n', n) \in E\}$*, and* succ$[n] = \{n' : (n, n') \in E\}$.

All nodes of a control flow graph are reachable from the initial node, i.e. there is a path from n_0 to each node n. path$[n]$ is the set of all paths from the initial node to n. path$[n)$ is the set of paths from n_0 up to all predecessors of n.

The data flow information is represented as a semi-lattice:

Definition 2 *A* semi-lattice $(L, \sqcap)$ *is a set L with a binary* meet *operation $\sqcap$ such that for all $a, b, c \in L$ the following holds:*

$$\begin{array}{rcll} a \sqcap a & = & a & \textit{Idempotent} \\ a \sqcap b & = & b \sqcap a & \textit{Commutative} \\ a \sqcap (b \sqcap c) & = & (a \sqcap b) \sqcap c & \textit{Associative} \end{array}$$

For two elements $a, b \in L$, we define:

$$\begin{array}{rcl} a \sqsubseteq b & \Leftrightarrow & a \sqcap b = a \\ a \sqsubset b & \Leftrightarrow & a \sqcap b = a \textit{ and } a \neq b \\ a \sqsupseteq b & \Leftrightarrow & a \sqcap b = b \\ a \sqsupset b & \Leftrightarrow & a \sqcap b = b \textit{ and } a \neq b \end{array}$$

$(L, \sqcap)$ has a zero-element $\perp$ *(bottom), if $\forall x \in L : x \sqcap \perp = \perp$ and a* one-element $\top$ *(top), if $\forall x \in L : x \sqcap \top = x$. From now on we assume that $(L, \sqcap)$ has a zero-element, but not necessary a one-element. We can extend the $\sqcap$ operation:*

$$\prod_{i=1}^{n} x_i = x_1 \sqcap x_2 \sqcap \ldots \sqcap x_n \textit{ with } \prod_{x \in \emptyset} = \top$$

A sequence of elements $x_1, x_2, \ldots, x_n$ of L is called a chain*, if $\forall 1 \leq i < n : x_i \sqsupset x_{i+1}$. $(L, \sqcap)$ is called* bounded *if for all $x \in L$ there is a constant b_x such that each chain starting with x has length at most b_x. If $(L, \sqcap)$ is bounded, we can define for each countably infinite set S of elements of L: $\prod_{x \in S} x = \lim_{n \to \infty} \prod_{i=1}^{n} x_i$. Since S is bounded, there is a number m with $\prod_{x \in S} x = \prod_{i=1}^{m} x_i$*

How a single program statement transforms, by its symbolic execution, the data flow information valid before its execution, is described by a *transfer function*:

Definition 3 *Let $(L, \sqcap)$ be a bounded semi-lattice. A set F of functions on L is called an* monotone function space associated with L, *if the conditions [M1] – [M4] are satisfied. If also [M5] is valid, it is called a* distributive function space associated with L.

- [M1] *All functions $f \in F$ are monotone: $\forall x, y \in L : f(x \sqcap y) \sqsubseteq f(x) \sqcap f(y)$. This is equivalent to: $\forall x, y \in L : x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$.*
- [M2] *There is an identity function* id $\in F$ *with: $\forall x \in L :$* id$(x) = x$.
- [M3] *F closed under composition: $\forall f, g \in F : f \circ g \in F$.*
- [M4] *L is the closure of $\{\perp\}$ with respect to the $\sqcap$ operation and application of functions in F.*
- [M5] *All functions are distributive: $\forall x, y \in L : f(x \sqcap y) = f(x) \sqcap f(y)$.*

A *monotone data flow framework* is defined as:

Definition 4 *A* monotone data flow framework *is a triple $D = (L, \sqcap, F)$ where $(L, \sqcap)$ is a bounded semi-lattice and F is a monotone function space associated with L. An* instance *of a monotone data flow framework is a pair $I = (G, M)$ where $G = (N, E, n_0)$ is a control flow graph and $M : N \to F$ is a labeling which maps each node from N onto a function of F.*

If F is a distributive function space, D is called a distributive data flow framework.

The "maximal (or worst case) information reaching a program statement" is given by the following

Definition 5

$$\prod_{p \in \mathrm{path}[n)} f_p(\perp)$$

is called the meet over all path *and represents the "maximal (or worst case) information reaching a node n of the program". f_p is the transfer function of the path p (see below).*

5 Properties of Some DFA Frameworks

First we give some properties of bit-vector data flow frameworks $\mathcal{D}^{\mathcal{B}}$ which is then generalized to $\mathcal{D}^{\mathcal{C}}$. At the end of this section then we apply these results to the transfer functions of statements and statement sequences.

5.1 Properties of the Boolean Semi-Lattice

Since we restrict our investigation to the class of bit-vector data flow problems, we give some general results for the boolean semi-lattice:

Definition 6 *The data flow information of an entity is a value of the set $\mathcal{B}$ (Bool) $\mathcal{B} = \{\top, \perp\}$. For a given binary meet operation $\sqcap$, $\perp \sqsubset \top$ must hold.*

Observation 1: Obviously there are only two different binary operations which can be the *meet* operation of a semi-lattice: $\wedge$ (*boolean and*) and $\vee$ (*boolean or*)[6]. They are given as shown in Table 1.

			$\sqcap = \wedge$, $\top =$ TRUE			$\sqcap = \vee$, $\top =$ FALSE		
a	b	$a \sqcap b$	a	b	$a \wedge b$	a	b	$a \vee b$
$\top$	$\top$	$\top$	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE
$\top$	$\perp$	$\perp$	TRUE	FALSE	FALSE	FALSE	TRUE	TRUE
$\perp$	$\top$	$\perp$	FALSE	TRUE	FALSE	TRUE	FALSE	TRUE
$\perp$	$\perp$	$\perp$	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE

Table 1: The boolean meet operations.

Observation 2: There are only four functions $\mathcal{B} \to \mathcal{B}$: the two constant functions, the identity, and negation.

use	u($\top$)	$= \top$	u($\perp$)	$= \top$
modify	m($\top$)	$= \perp$	m($\perp$)	$= \perp$
identity	id(x)	$= x$		
negation	$\overline{\top}$	$= \perp$	$\overline{\perp}$	$= \top$

[6]The other possible 14 binary operations over $\mathcal{B}$ do not have the required properties, even if some may have an interesting interpretation such as *xor:* the data flow information is valid, if it is valid in exact one path.

Obviously, the negation function is not monotone and not distributive. The other three are both monotone and distributive. Often, the constant functions are interpreted, respectively as *use*, which generates or uses some information, and *modify*, which modifies or invalidates it.

We finish this section with:

Observation 3: For any two-element semi-lattice $(\mathcal{B}, \sqcap)$, there is exactly one monotone function space $\mathcal{F}^{\mathcal{B}} =_{def} \{\mathrm{u}, \mathrm{m}, \mathrm{id}\}$ associated with $\mathcal{B}$[7]. It is also distributive. $\mathcal{D}^{\mathcal{B}} = (\mathcal{B}, \sqcap, \mathcal{F}^{\mathcal{B}})$ is called the *one bit data flow analysis framework*. There are only two interpretations of the meet operation: the boolean $\wedge$ and $\vee$ operation, respectively. Their DFA interpretation is: the information valid before a node n must be valid on all ($\wedge$) (at least one ($\vee$)) path reaching n.

5.2 Properties of the Function Space $\mathcal{F}^{\mathcal{C}}$

A slight generalization of the $\mathcal{D}^{\mathcal{B}}$ DFA framework is $\mathcal{D}^{\mathcal{C}}$, for which the following lemma obviously holds:

Lemma 1 *Let $(\mathcal{C}, \sqcap)$ be a bounded semi-lattice, and $\mathcal{F}^{\mathcal{C}}$ a set of functions $\mathcal{C} \rightarrow \mathcal{C}$, such that $\mathcal{F}^{\mathcal{C}}$ contains only the identity function* id *and for each element c of $\mathcal{C}$ its constant function* const_c *with $\forall x \in \mathcal{C} : \mathrm{const}_c(x) = c$. Then $\mathcal{F}^{\mathcal{C}}$ is a distributive function space, and the corresponding* constant data flow framework *$\mathcal{D}^{\mathcal{C}}$ is distributive.*

We consider now "composition chains" of functions $f_n \circ f_{n-1} \circ \cdots \circ f_1(x)$ and show some properties. The following lemmata helps us to compute the data flow information which is valid after a `PAR` statement: if for such a composition chain a predicate P holds for all $x \in C$, then there is a function f_i in it, such that P holds for all x, and all "following" $f_j, j > i$ do not invalidate the predicate.

Lemma 2 *Let $f_1, \cdots, f_n \in \mathcal{F}^{\mathcal{C}}$ and P a predicate over $\mathcal{C}$.*

$$\forall x \in \mathcal{C} : P[f_n \circ f_{n-1} \circ \cdots \circ f_1(x)]$$
$$\textit{iff}$$
$$\exists 1 \leq i \leq n : \forall x \in \mathcal{C} : P[f_i(x)] \textit{ and}$$
$$\forall i < j \leq n : \forall x \in \mathcal{C} : P[f_j(f_i(x))]$$

The lemma can be proved using induction over the number n of functions in the composition chain. The next lemma is a corollary of the previous one:

Lemma 3 *Let $f_1, \cdots, f_n \in \mathcal{F}^{\mathcal{C}}$. Then: $\exists 1 \leq i \leq n : \forall x \in \mathcal{C} : f_n \circ \cdots \circ f_1(x) = f_i(x)$ and $\forall i < j \leq n : f_j = \mathrm{id}$*

The next lemmata state that under some circumstances the order of the functions of a composition chain may be changed and still return the same value.

Lemma 4 *Let $f_1, \ldots, f_n \in \mathcal{F}^{\mathcal{C}}$. From $\exists 1 \leq i \leq n : \forall x \in \mathcal{C} : f_n \circ f_{n-1} \ldots \circ f_1(x) = f_i(x)$, it follows that for an arbitrary permutation $(k_1, \ldots, k_{i-1})$ of the numbers $1, \ldots, i-1$ holds:*

$$f_n \circ f_{n-1} \circ \ldots \circ f_1(x) = f_n \circ f_{n-1} \circ \ldots f_i \circ f_{k_{i-1}} \circ f_{k_{i-2}} \circ f_{k_1}(x)$$

Proof (Lemma 4) If f_i is a constant function, it returns the same result for all arguments. Hence the order of the functions which form the argument of f_i is not important. If f_i is the identity function, all other functions must also be the identity function, otherwise $f_n \circ f_{n-1} \ldots \circ f_1(x) = f_i(x)$ would not hold for all x. ∎

[7]Note: To be a monotone function space associated with $\mathcal{B}$, all three functions are needed.

The following lemmata state some properties of the value of composition chains. They answer the question which information is valid before a statement inside a `PAR` statement.

Lemma 5 *Let $f, g \in \mathcal{F}^{\mathcal{C}}$. Then for arbitrary $x \in \mathcal{C}$:*

$$x \sqcap f(x) \sqcap g(f(x)) = x \sqcap f(x) \sqcap g(x)$$

Proof (Lemma 5) If $g = \mathrm{const}_c$ then: $x \sqcap f(x) \sqcap g(f(x)) = x \sqcap f(x) \sqcap g(x)$.

If $g = \mathrm{id}$ then: $x \sqcap f(x) \sqcap g(f(x)) = x \sqcap f(x) = x \sqcap f(x) \sqcap g(x)$. ∎

Using induction over the number n of functions in the composition chain we can conclude:

Lemma 6 *Let $f_1, \ldots, f_n \in \mathcal{F}^{\mathcal{C}}$. Then for arbitrary $x \in \mathcal{C}$:* $x \sqcap f_1(x) \sqcap f_2 \circ f_1(x) \sqcap \ldots \sqcap f_n \circ f_{n-1} \ldots \circ f_1(x) = x \sqcap f_1(x) \sqcap f_2(x) \sqcap \ldots \sqcap f_n(x)$

5.3 Properties of Statement Sequences and Composition Chains

From now on, we consider only $\mathcal{D}^{\mathcal{C}}$. We now connect the functions to a statement sequence, which represents an execution path. Then we state properties of the function composition chains, if two (or more) statement sequences are executed in parallel. This is modeled by considering the set of topological sortings of the statements contained in the sequences.

Definition 7 *Let $s_1, \ldots, s_n$ be simple statements, such as assignments, which are executed in the order $p \equiv \langle s_1; \ldots; s_n \rangle$. Let $f_{s_i} \in \mathcal{F}^{\mathcal{C}}$ be the transfer function connected to the statement s_i. $f_p(x) =_{def} f_{s_n} \circ f_{s_{n-1}} \circ \ldots \circ f_{s_1}(x)$*

Definition 8 *Let $s'_1, \ldots, s'_{n'}$ and $s''_1, \ldots, s''_{n''}$ be simple statements, which are executed in the order $p' \equiv \langle s'_1; \ldots; s'_{n'} \rangle$ and $p'' \equiv \langle s''_1; \ldots; s''_{n''} \rangle$, respectively.* $\mathrm{TopSorts}(p', p'')$ *is the set of statement sequences which result from a topological sorting of the two sequences p' and p''. For two statements s'_i, s''_j no order is defined, and s'_i must be executed before s'_{i+1} (also for the statements s''_j).*

Lemma 7 *Let $s'_1, \ldots, s'_{n'}$ and $s''_1, \ldots, s''_{n''}$ be simple statements, which are executed in the order $p' \equiv \langle s'_1; \ldots; s'_{n'} \rangle$ and $p'' \equiv \langle s''_1; \ldots; s''_{n''} \rangle$, and $f_{s'_i}, f_{s''_j} \in \mathcal{F}^{\mathcal{C}}$ the functions corresponding to the statements s'_i and s''_j, respectively. The following holds:*

$$\bigsqcap_{p \in \mathrm{TopSorts}(p', p'')} f_p(x) = f_{p';p''}(x) \sqcap f_{p'';p'}(x)$$

Where $p; q$ is the concatenation of two statement sequences p and q.

Proof (Lemma 7) We prove this lemma in several steps:

1. Let $p \equiv \langle s_1; \ldots; s_n \rangle \in \mathrm{TopSorts}(p', p'')$. With lemma 3 we have a $1 \leq i \leq n$ where $f_p(x) = f_i(x)$[8] and $\forall i < j \leq n : f_j = \mathrm{id}$. If there are several such i, we use the largest one. Now s_i, the statement determining the value of the path p, may be contained either in p' or p''.

[8]f_i is an abbreviation of f_{s_i}.

2. The set $\text{TopSorts}(p', p'')$ can be split into two disjoint subsets seq' and seq'', where:
   $seq' =_{def} \{q \in \text{TopSorts}(p', p'') \mid \forall x : f_q(x) = f_i(x) \text{ and } s_i \in p'\}$, i.e. seq' contains all those paths, whose value is determined only by statements contained in p'. seq'' is defined analogously.

   Since $\text{TopSorts}(p', p'') = \text{seq}' \cup \text{seq}''$, it follows that $\bigsqcap_{p \in \text{TopSorts}(p',p'')} f_p(x) = \bigsqcap_{p \in \text{seq}'} f_p(x) \sqcap \bigsqcap_{p \in \text{seq}''} f_p(x)$.
3. Proposition: If $s_i \in p''$, then for all x:
   $f_p(x) = f_{p';p''}(x)$.

   Proof: The proposition is proved, by reordering the sequence p stepwise: The statements $s_{k_1}, s_{k_2} \in p$ with $1 \leq k_1, k_2 < i$ may be reordered in a way that all statements $s_{k_1} \in p'$ are placed before $s_{k_2} \in p''$ and still fulfill the order constraints of p' and p'', respectively. Lemma 4 guarantees that the value of this reordered sequence is still equal to $f_p(x)$.

   The instructions $s_k \in p'$ with $k > i$ may also be placed before s_i, since $f_k = \text{id}$.

   The statements $s_k \in p''$ with $k > i$ need not be reordered.

   Hence if $s_i \in p''$ then $f_p(x) = f_{p';p''}(x)$. Analogously, if $s_i \in p'$ the $f_p(x) = f_{p'';p'}(x)$.
4. Now the statement sequences from seq' and seq'' may be reordered as shown above, while not changing their value. Hence $\bigsqcap_{p \in \text{seq}'} f_p(x) = f_{p'';p'}(x)$ and $\bigsqcap_{p \in \text{seq}''} f_p(x) = f_{p';p''}(x)$. And so:
   $\bigsqcap_{p \in \text{TopSorts}(p',p'')} f_p(x) = f_{p'';p'}(x) \sqcap f_{p';p''}(x)$.

■

6 Data Flow Analysis of the PAR Statement

We now solve the data flow analysis problem for the PAR statement in two steps:

1. which information is valid before each statement in a process body, and
2. which data flow information is valid after the PAR statement?

But before doing this, we need some more definitions.

6.1 Statement Traces Generated by the PAR Statement

If the maximum (worst case) data flow information of some statements is given as the *meet over all paths*, the question arises: how can we compute all paths of a parallel program?

We now consider *statement traces* instead of paths in the control flow graph; this simplifies the presentation of the next results.

Definition 9 *A* statement trace *or* statement execution sequence *is a list of simple statements in the order they are executed by a single run of a program. The set* $\text{seq}[S]$ *is the set of all traces generated by statement* S. $s \in p \in \text{seq}[S]$ *means that* s *is a simple statement contained in sequence* q.

It is well known how the set of traces is constructed for sequential programs:

- For the concatenation of statements $S_1; S_2$[9] the traces produced from S_1 and S_2 are concatenated.
- For an IF statement, the traces generated by the THEN and ELSE parts are united.
- For loops, all n-fold concatenations of the traces produced by the loop body are united.

Since we assumed an interleaving semantics for the execution of the statements of our PAR statement, it is obvious that the topological sorting of the traces produced by the process bodies, determines all traces of the PAR statement.

Definition 10 *For a given trace p the set* $\text{prefixes}(p)$ *is the set of all prefixes of p. It includes the empty trace and the entire trace p.* $\text{prefixes}(P)$ *is the extension to a set of traces P, so that it contains all prefixes of all traces of* $p \in P$. *For a statement S,* $\text{prefixes}[S]$ *is the abbreviation of* $\text{prefixes}(seq[S])$.

Definition 7 is now extended for arbitrary (composite) statements:

Definition 11 *Let* $S, S_1, \ldots, S_n$ *be arbitrary (composite) statements:* $f_S(x) =_{def} \bigsqcap_{p \in \text{seq}[S]} f_p(x)$.

$f_{S_1;S_2;\ldots;S_n}(x) =_{def} f_{S_n} \circ f_{S_{n-1}} \circ \ldots \circ f_{S_1}(x)$ *is the function corresponding to the statement sequence* $S_1; \ldots; S_n$.

an obvious lemma is:

Lemma 8 *Let* $S_1, \ldots, S_n$ *be arbitrary statements and let be* $S \equiv S_1; \ldots; S_n$. *Then for distributive functions* $f \in F$: $f_{S_1;S_2;\ldots;S_n}(x) = f_S(x)$, *while for monotone* $f \in F$, *only* $f_{S_1;S_2;\ldots;S_n}(x) \sqsubseteq f_S(x)$ *holds.*

Definition 12 *Let s be a statement inside a (nested)* PAR *statement.* $\text{sibl}[s]$ *is the set of all simple statements which could possibly be executed in parallel to s. If s is part of a replicated process body, all statements of this process body are also contained in* $\text{sibl}[s]$.

Note that the property $s' \in \text{sibl}[s]$ of s' is a purely syntactical one, which can easily be determined from the source program.

6.2 Which Data Flow Information Reaches a Statement

Let s be a statement inside a PAR statement. By definition the information valid before s is given by the meet over all traces reaching s. To answer the question which these traces are, let us examine the following example[10]: PAR $s \mid s_1; s_2; \ldots; s_n$ END. s may now be the first statement executed in an interleaving produced by this PAR statement. On the other hand, s_1 may be executed before s in another interleaving, or $s_1; s_2$ are executed before s, etc. Hence the set of statements executed before s is given by $\text{prefixes}(\langle s_1; \ldots; s_n \rangle)$. Using lemma 6 we can conclude that: $\bigsqcap_{p \in \text{prefixes}(\langle s_1;\ldots;s_n \rangle)} f_p(x) = x \sqcap \bigsqcap_{i=1}^{n} f_{s_i}(x)$. If the process body S of PAR $s \mid S$ END which is executed in parallel to s produces more than one execution sequence, the set of execution sequences reaching s is given by: $\bigcup_{p \in \text{seq}[S]} \text{prefixes}(p)$, and hence: $\bigsqcap_{p \in \text{prefixes}(\text{seq}[S])} f_p(x) = x \sqcap \bigsqcap_{t \text{ is a statement from } S} f_t(x)$.

The fact that there may be statements, which are *always* executed before s, does not influence our considerations, since the argument x of the function f_s reflects the

[9]Capital letters S denote composite statements, whereas small letters s denote simple ones.

[10]Note that in these traces some statement within the body of the PAR statement may not be included, since they are executed "after" s.

information reaching s, if no parallel statements are executed before s.

Since PAR statements may be nested, the following theorem is a consequence of the above:

Theorem 1 *Let s be a simple statement in a program, and* $\mathrm{sibl}[s]$ *the set of simple statements possibly executed in parallel to s. Then for $\mathcal{D}^{\mathcal{C}}$ data flow problems, the following information is valid before s:*

$$x \sqcap \prod_{t \in \mathrm{sibl}[s]} f_t(x),$$

where x is the information valid before the PAR *statement.*

If we use the following definition, we can restate the theorem so that it is easier to use as a base for an implementation.

Definition 13 *For statement s, we define* $\mathrm{in}^{;}[s]$ *as the information reaching s on a "sequential" trace from the program entry. That is, none of siblings of s are executed before s. And let* $\mathrm{in}^{||}[s]$ *be the information reaching s if all possible trace are considered.*

If a function $f_t \in \mathcal{F}^{\mathcal{C}}, t \in \mathrm{sibl}[s]$ is a constant function, then the value of $f_t(x)$ is independent of x: $f_t(x) = f_t$. Otherwise f_t is the identity, and $f_t(x) = x$. Then $\mathrm{in}^{;}[s]$ is simply the value of x, and the Theorem 1 can be restated as:

$$\mathrm{in}^{||}[s] = \mathrm{in}^{;}[s] \sqcap \prod_{t \in \mathrm{sibl}[s]} f_t \tag{1}$$

6.3 Which Information is Valid After the PAR Statement

Lemma 7 is the cornerstone for the following theorems. After extending it to sets of paths we obtain:

Theorem 2 *For the $\mathcal{D}^{\mathcal{C}}$ data flow problems and the* PAR S_1 | S_2 END *statement the following holds:*

$$\prod_{p \in \mathrm{seq}[\texttt{PAR } S_1 \texttt{ | } S_2 \texttt{ END}]} f_p(x) = \prod_{p \in \mathrm{seq}[S_1;S_2]} f_p(x) \sqcap \prod_{p \in \mathrm{seq}[S_2;S_1]} f_p(x)$$

To extend this theorem for PAR statements with more than two process bodies, we define:

Definition 14 *The set of* simple permutations s_perm *of the numbers $1, \ldots, n$ is given by:* $\mathrm{s\_perm}(1,n) =_{def}$ $\{\langle 1,2,\ldots,n-1,n\rangle, \langle n,2,\ldots,n-1,1\rangle,$ $\langle 1,n,3,\ldots,n-1,2\rangle, \ldots, \langle 1,2,3,\ldots,n,n-1\rangle\}$. *That is, the i^{th} number is exchanged with the last number in the sequence $\langle 1,2,\ldots,n\rangle$. $\vec{\imath} \equiv \langle i_1, i_2, \ldots, i_n\rangle$ denotes an element of this set. Note that* $\mathrm{s\_perm}(1,n)$ *has only n elements.*

If we have a closer look at the proof of lemma 7, we see that the order of the statements s_k, with $k < i$ has no influence on that result. Hence we have the following lemma:

Lemma 9 *Let $p_1, \ldots, p_n$ statement sequences of simple statements. Then:*

$$\prod_{p \in \mathrm{TopSorts}(p_1,\ldots,p_n)} f_p(x) = \prod_{\vec{\imath} \in \mathrm{s\_perm}(1,n)} f_{p_{i_1};\ldots;p_{i_n}}(x)$$

Note that any other permutation would serve, as long as each statement sequence appears at least once at the end. Hence we have the following theorem:

Theorem 3 *For the $\mathcal{D}^{\mathcal{C}}$ data flow problems and the* PAR S_1 | ... | S_n END *statement:*

$$\prod_{p \in \mathrm{seq}[\texttt{PAR } S_1 \texttt{ | ... | } S_n \texttt{ END}]} f_p(x) = \prod_{\vec{\imath} \in \mathrm{s\_perm}(1,n)} f_{S_{i_1};\ldots;S_{i_n}}(x).$$

It is easy to see that the result is not changed by a replicated process body `[var := lwb TO upb]` S with `upb - lwb + 1 > 0`, since for the result only S is important. If `upb - lwb + 1` $\geq$ `0`, then it may happen that S is never executed, and the theorem should be adjusted accordingly. We will see in the implementation section 8, how this could easily be done.

7 Bit-Vector Implementation

From now on, we consider only the $\mathcal{D}^{\mathcal{B}}$ data flow analysis framework.

Until now we considered only one program entity, such as a single program variable or a single expression. When implementing data flow analysis, usually all entities are considered at the same time. Hence we are dealing with sets of informations, valid at a program point. Each entity is coded by a number $1 \ldots |\mathrm{entities}|$. For the class of one-bit data flow problems $\mathcal{D}^{\mathcal{B}}$ there is a quite efficient implementation of sets: the bit-vector.

Definition 15 *A bit-vector is the characteristic function* vec *of a finite set of of object numbers $1 \ldots n = |\mathrm{entities}|$.* $\mathrm{vec} : \{1 \ldots n\} \rightarrow \{\mathrm{TRUE}, \mathrm{FALSE}\}$ *with $1 \leq n$. Usually* $\mathrm{vec}(i)$ *is denoted as* $\mathrm{vec}[i]$.

The set operations $\cup, \cap, \overline{}$ are defined for bit-vectors, by element wise application of the boolean operations $\vee, \wedge, \overline{}$. The set difference is defined by $a - b =_{def} a \cap \overline{b}$.

The empty set $\emptyset$ is represented by the bit-vector, in which all values are FALSE.

Usually for each statement and basic block the following four sets are defined [1]: *gen*, *kill*, *in* and *out*. *gen* (*kill*) is the set of information generated (invalidated) by this statement/basic block. *in* is the set of information valid immediately before execution of this statement and *out* the information valid immediately after that point.

As seen before, the $\sqcap$ operation is either set union or set intersection. The DFA problems using set union (intersection) as meet operations are called *may problems* (*must problems*), since the information must reach a given program point on at least one (on all) paths leading to that point.

We define now the four sets in terms of the transfer functions. Each statement s has a separate transfer function for each entity e, denoted by f_s^e.

Definition 16 *Let s be a statement,*
$\mathrm{gen}_s[e] = \mathrm{TRUE}$ *iff* $f_s^e = \mathrm{u}$ *(use),*
$\mathrm{kill}_s[e] = \mathrm{TRUE}$ *iff* $f_s^e = \mathrm{m}$ *(modify),*
$\mathrm{in}_s[e] = \mathrm{TRUE}$ *iff* $\prod_{p \in \mathrm{path}(s)} f_p^e(\perp) = \top$ *and*
$\mathrm{out}_s[e] = \mathrm{TRUE}$ *iff* $\prod_{p \in \mathrm{path}[s]} f_p^e(\perp) = \top$.
Obviously: $\mathrm{gen}_s \cap \mathrm{kill}_s = \emptyset$

We will first formulate the result of Theorem 1 in the form of Equation 1 using the DFA sets. Note that we are now using $\mathcal{D}^{\mathcal{B}}$:

In the equation $\mathrm{in}^{\|}{}_s[e] = \mathrm{in}^{;}{}_s[e] \sqcap \prod_{t\in \mathrm{sibl}[s]} f_t^e$, all f_t are constant functions. The right hand side is $\top$, iff both parts of it evaluate to $\top$. The following equivalences hold:
$\prod_{t\in \mathrm{sibl}[s]} f_t^e = \top \Leftrightarrow \forall t \in \mathrm{sibl}[s] : f_t^e = \top \Leftrightarrow$
$\nexists t \in \mathrm{sibl}[s] : f_t^e = \bot \Leftrightarrow \bigwedge_{t\in \mathrm{sibl}[s]} \mathrm{gen}_s[e] = \mathrm{TRUE} \Leftrightarrow$
$\bigvee_{t\in \mathrm{sibl}[s]} \mathrm{kill}_s[e] = \mathrm{FALSE}$.

We now distinguish:

$\sqcap = \wedge; \top = \mathrm{TRUE}$

$$\prod_{t\in \mathrm{sibl}[s]} f_t^e = \top \Leftrightarrow \bigwedge_{t\in \mathrm{sibl}[s]} f_t^e = \mathrm{TRUE} \Leftrightarrow$$
$$\overline{\bigvee_{t\in \mathrm{sibl}[s]} \mathrm{kill}_s[e]} = \mathrm{TRUE}$$

$\sqcap = \vee; \top = \mathrm{FALSE}$

$$\prod_{t\in \mathrm{sibl}[s]} f_t^e = \top \Leftrightarrow \bigvee_{t\in \mathrm{sibl}[s]} f_t^e = \mathrm{FALSE} \Leftrightarrow$$
$$\overline{\bigwedge_{t\in \mathrm{sibl}[s]} \mathrm{gen}_s[e]} = \mathrm{FALSE} \Leftrightarrow$$
$$\bigvee_{t\in \mathrm{sibl}[s]} \overline{\mathrm{gen}_s[e]} = \mathrm{FALSE} \Leftrightarrow$$
$$\bigvee_{t\in \mathrm{sibl}[s]} \mathrm{gen}_s[e] = \mathrm{TRUE}.$$

And we can state the following theorem:

Theorem 4 *The information of a one-bit DFA problem reaching a statement s can be computed by:*

$$\mathrm{in}^{\|}{}_s = \begin{cases} \mathrm{in}^{;}{}_s - \bigcup_{t\in \mathrm{sibl}[s]} \mathrm{kill}_t & \textit{for} \quad \sqcap = \wedge \\ & \textit{with} \quad \mathrm{in}^{;}{}_{s_0} = \top = \mathrm{TRUE} \\ \mathrm{in}^{;}{}_s \cup \bigcup_{t\in \mathrm{sibl}[s]} \mathrm{gen}_t & \textit{for} \quad \sqcap = \vee \\ & \textit{with} \quad \mathrm{in}^{;}{}_{s_0} = \top = \mathrm{FALSE} \end{cases}$$

(s_0 is the first statement of the program).

The next step is the adaptation of Theorem 3. Before doing so, we restate the equations for in and out in the sequential case, as they may be found e.g. in [1]:
For a simple statement the relation is given by:

$$\mathrm{out}_s = \mathrm{gen}_s \cup \mathrm{in}_s - \mathrm{kill}_s \tag{2}$$

For sequential composition of statements $s \equiv s_1; s_2$:

$$\mathrm{out}_s = \mathrm{out}_2 \cup \mathrm{out}_1 - \mathrm{kill}_2 \tag{3}$$

For branches in a sequential program:

$$\begin{aligned} \mathrm{in}_s &= \prod_{p\in \mathrm{pred}[s]} \mathrm{out}_p \\ \mathrm{out}_s &= \mathrm{gen}_s \cup \mathrm{in}_s - \mathrm{kill}_s \end{aligned} \tag{4}$$

Using these equations we can compute the right hand side of the equation from Theorem 3: $\prod_{\vec{\imath}\in \mathrm{s\_perm}(1,n)} f_{S_{i_1};\ldots;S_{i_n}}(x)$. We again have to distinguish between may and must problems. Hence the following theorem can be given:

Theorem 5 *The information* out_S *of a one-bit DFA problem valid after the* `PAR` S_1 | ... | S_n `END` *statement S can be computed as:*

$$\mathrm{out}_S = \begin{cases} \left(\left(\bigcup_{i=1}^n \mathrm{gen}_i\right) - \left(\bigcup_{i=1}^n \mathrm{kill}_i\right)\right) \cup \left(\mathrm{in}^{;}{}_S - \bigcup_{i=1}^n \mathrm{kill}_i\right) \\ \qquad \textit{for } \sqcap = \wedge \\ \bigcup_{i=1}^n \mathrm{gen}_i \cup \left(\mathrm{in}^{;}{}_S - \bigcup_{i=1}^n \mathrm{kill}_i\right) \qquad \textit{for } \sqcap = \vee \end{cases}$$

where $\mathrm{in}^{;}{}_S$ *is the information valid before the* `PAR` *statement, and* gen_i *and* kill_i *are the sets corresponding to the process bodies* S_i.

Now we have to determine which information is generated and invalidated by a `PAR` statement as a whole. Again we start with the sequential composition of statements (following [1]): For $S \equiv S_1; S_2$ we have:

$$\begin{aligned} \mathrm{gen}_S &= \mathrm{gen}_2 \cup \mathrm{gen}_1 - \mathrm{kill}_2 \\ \mathrm{kill}_S &= \mathrm{kill}_2 \cup \mathrm{kill}_1 - \mathrm{gen}_2 \end{aligned} \tag{5}$$

Hence we can follow the same arguments as for $\mathrm{out}_{\mathrm{PAR}}$, with the simplification that the term in $-(\mathrm{kill} \cup \ldots)$ does not exist. We note here that if the "In-Out-problem[11]" is a may-problem, then the "Gen-Problem[12]" is a may-problem too, while the corresponding "Kill-problem" is a must-problem, and vice versa, if the In-Out-problem is a must-problem. So the next theorem can be stated:

Theorem 6 *The information of a one-bit DFA problem generated and invalidated by a*
`PAR` S_1 | ... | S_n `END` *statement S can be computed as:*

$$\begin{aligned} \textit{In-Out-problem } \sqcap = \wedge : \mathrm{gen}_S &= \left(\bigcup_{i=1}^n \mathrm{gen}_i\right) - \left(\bigcup_{i=1}^n \mathrm{kill}_i\right) \\ \mathrm{kill}_S &= \bigcup_{i=1}^n \mathrm{kill}_i \\ \textit{In-Out-problem } \sqcap = \vee : \mathrm{gen}_S &= \bigcup_{i=1}^n \mathrm{gen}_i \\ \mathrm{kill}_S &= \left(\bigcup_{i=1}^n \mathrm{kill}_i\right) - \left(\bigcup_{i=1}^n \mathrm{gen}_i\right) \end{aligned}$$

Having all these theorems, we see that we have avoided the state-space explosion problem.

These results are given for an "isolated" `PAR` statement. The next section will put them in the context of a *parallel control flow graph* and we will see how this gives us an elegant algorithm for computing the data flow information of a parallel program.

8 The Parallel Control Flow Graph

The following explanation is based on our implementation of the parallel language *Modula-P* [22] developed in the COMPARE project. To express parallelism in the intermediate language *CCMIR-P*[13] we define some additional CCMIR statements and define a *parallel control flow graph* (PCFG).

Looking at the results of this theory, we observe that the `PAR` statement and its processes are treated by it as a single statement with a complex behavior. The idea for integrating the analysis of a `PAR` statement is to treat it like other CCMIR statements such as assignment or procedure

[10]The symbol X_{s_i} is abbreviated to X_i.

[11]Which information is valid before/after a statement.

[12]Which information is generated by the statement.

[13]*Common* COMPARE *Medium Intermediate Representation, Parallel extension*; the intermediate language of the COMPARE compilers.

```
REPEAT ..;
  IF ..
  THEN
    x:=..;
    PAR
      ..; PAR Body1a — Body1b END; ..;
     — Body2 — Body3
    END
    p (..); z := f (..);
    PAR Body4a — Body4b END;
  ELSE ..
  END;
UNTIL ..
```

Table 2: A program fragment with nested PAR's.

call, except that it has a more complex DFA behavior. The DFA effects of the `PAR` statement are determined solely by its process bodies.

The central idea of our parallel control flow graph (PCFG) is that it is a forest of disjoint CFG's. Each process body and procedure body constitutes a separate CFG. Since jumps into or out of process bodies are forbidden in the source language, there are no *jump edges* connecting the CFG's.

To form a PCFG of a procedure we connect these separate CFG's by adding *parallel edges* between the CFG of a process body and the `mirParallel` statement containing this process body.

For the program fragment shown in Table 2 the PCFG is given in Figure 1.

After parsing a source program, we obtain a list of *all* basic blocks, constituting an entire procedure. There is no specific order in this list. From that, we compute for each basic block the list of predecessors and successors (cf. [1]). Each basic block has zero, one or two successors. It has none, if it has as its last statement the `EndProcedure` or `EndProcess` statement. It has one, if the last statement is a `mirGoto`, and two if this is a `mirIf` statement. Entry basic blocks are marked by the `BeginProcedure` and `BeginProcess` CCMIR instructions: their basic blocks have no predecessors. To find the roots of the CFG's we simply scan the procedure's list of all basic blocks for basic blocks having no predecessors.

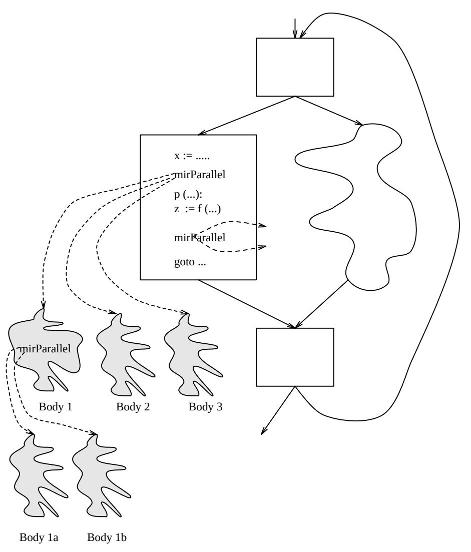
If a replicator specifies that no process should be created, then we draw an extra jump edge from the process body entry to its exit. This solves the problems mentioned in the note to Theorem 3.

Our definition of a PCFG differs from the one given in [26] in the way that their PCFG has two kinds of nodes: "ordinary" basic blocks and *super nodes (or parallel blocks)*. Such a super node represents the entire process body.

9 Solving the Data Flow Equations of Concurrent Programs

Let's assume we do not have nested `PAR` statements. Then we must analyze a program in the following order:

1. Compute *gen* and *kill* information for all process bodies and the `mirParallel` statement itself.
2. Compute *gen* and *kill* information for all statements of the procedure's CFG.
3. Compute *in* and *out* for all statements of the statements of the procedure's CFG.



The dashed lines are parallel edges connecting the separate CFG's of the process bodies to their `mirParallel` statement. The grey boxes are CFG's of the process bodies. The other boxes are part of the procedure body's CFG.

Figure 1: A PCFG for the program fragment of Table 2.

4. Since we know the exact information reaching the `mirParallel` statement, we can compute the *in* and *out* information of the valid at the statements of the process bodies: the information reaching the `mirParallel` statement reaches the entry of each process body.

Now it is obvious how to deal with nested `PAR` statements:

1. Visit the deeply-nested process bodies first and compute their *gen* and *kill* information. This corresponds to a depth-first traversal of the PCFG along the parallel edges. This is called *inside-out computation of Gen/Kill.*
2. Compute *gen* and *kill* of all other statements of the procedure's CFG.
3. Compute *in* and *out* for all statements of the procedure's CFG.
4. Visit the `mirParallel` statements from *outside-in* (the reverse order of inside-out) and compute *in* and *out* of the process bodies. Outside-in is the top-down traversal of the PCFG along the parallel edges.

This kind of computation of the data flow information is a mixture of the structural [2], [1, page 611] (for the effect of the `mirParallel` statement) and iterative (all other statements) method. A similar idea has been presented for DFA of sequential problems by [8].

The computation of the *in* and *out* information must be done using an iterative algorithm [11, 1]. At a first glance the same seems to be true for the computation of *gen* and *kill*[14]. But this iterative approach for *gen* and *kill* solves a broader problem: it computes for each basic block *b* the set

[14]Note: We have to compute the *gen* and *kill* info for a set of basic blocks. In the sequential DFA, this information is not needed.

of *gen* and *kill* information reaching *b*. But we need only the *gen* and *kill* information which is valid at the end of a process body. We can therefore use a simpler algorithm which combines the computation of the local *gen* and *kill* with the computation of the *gen* and *kill* of a set of basic blocks. We visit the basic blocks in the reverse depth first order, which guarantees that at the end of the process we have the same result than using the iterative method.

10 Complexity of this DFA Algorithm

To estimate the complexity of this algorithm, we use as complexity measure the number of visits of a basic block during the iterative computation of the DFA information. A "comparable" sequential program is one where the `PAR` statement and its process bodies are executed sequentially.

For the computation of the *gen* and *kill* sets, we have to visit each basic block once, both in the sequential and parallel case.

During the computation of *in* and *out* we apply the iterative algorithm several times to different (and disjoint) sets of basic blocks: first, we compute the DFA information for the basic blocks of the procedure's CFG. Second, we compute *in* and *out* for the sets of basic blocks corresponding to process bodies. The process bodies are considered in the *outside-in* order of the `PAR` statement.

The number of iterations needed to compute the DFA information is determined by the *loop nested-ness* [7] of the source program.

Since we don't have jump edges between basic blocks of different process bodies, we are always computing the DFA information of disjoint sets of basic blocks. Hence the loop nested-ness is the same for the parallel and the comparable sequential program. Hence the overall number of basic block visits is equal in the parallel and the comparable sequential program.

As a result the data flow analysis of a parallel program has the same complexity as a comparable sequential one.

11 Current and Future Work

Currently the algorithm of [12] for elimination of partial redundancies is implemented in the COMPARE compiler for the source language *Modula-P* [22] which is an parallel extension of Modula-2.

12 Conclusion

This work shows that data flow analysis of parallel programs is possible, and can be done as efficiently as for sequential programs. The novelty is that there is no restriction in the kind shared memory access, nor in the "accuracy" of the resulting DFA information. Hence it is now possible to apply optimizing transformations, which are well known from the sequential context.

To show this, we proved some nice properties of the semilattice based data flow frameworks $\mathcal{D}^{\mathcal{C}}$ and $\mathcal{D}^{\mathcal{B}}$, which allowed us to reduce the number of interleavings needed for the computation of the *meet over all paths* solution of the DFA problem. Then we extended these results to bit-vectors, and obtained simple set equations, computing the DFA information valid inside and after a `PAR` statement. Based on that we gave a simple algorithm to compute the DFA information valid at all program points. This algorithm is a slight variant of the usual iterative DFA algorithm, but the basic blocks are visited in a special order: *inside-out and outside-in* of the nesting structure of the `PAR` statement.

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ldc	1, r0
ld	b, r1
st	r0, a
cmp	r1, 0
jeq	then_1
...	

3a

time	t_1	t_2	t_3	t_4	...	
Processor_1:	ldc 1, r0;	ld b, r1; *(r1 = 0)*	st r0, a;	cmp r1, 0;	...	$\textit{critical}_1$
Processor_2:	ldc 1, r0;	ld a, r1; *(r1 = 0)*	st r0, b;	cmp r1, 0;	...	$\textit{critical}_2$

3b

Table 3: Code and execution of reordered code

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A What Might Go Wrong: An Example

This section shows the potential problems, when applying sequential data flow analysis to an explicitly parallel program. And more that running an instruction scheduler may cause a wrong execution of a parallel program. The small program[16] given in Table 4 executes the processes P_1 and P_2 in parallel. It is intuitively clear that $\textit{critical}_1$ and $\textit{critical}_2$ are never executed at the same time.

[16] [13] presents this problem concerning the design of parallel computers.

On a first glance this program has a *race condition* [16], (both processes read and write the shared variables, without some kind of explicit synchronization) but this race condition is an intended one: The variables `a` and `b` are used to implement a simple synchronization scheme. Extensive work has been done on analyzing parallel programs for potential races but little work has been done to optimize them.

```
          a := 0; b := 0;
               PAR
      (P1)          |      (P2)
a := 1;             | b := 1;
IF b = 0            | IF a = 0
THEN critical1;     | THEN critical2;
       a := 0;      |        b := 0;
ELSE else1          | ELSE else2
END                 | END
               END
```

Table 4: Part of process synchronization code

A simple-minded optimizer could perform the following "optimizations" (which would be correct in sequential contexts):

- Propagate `a=0` and `b=0` to `IF a=0` and `IF b=0` respectively.
- Then the expressions could be statically evaluated to `TRUE`.
- Dead code elimination removes the `IF` and `ELSE` parts.

And as consequence, both, $\textit{critical}_1$ and $\textit{critical}_2$ would be executed.

But even without traditional optimizations performed by the compiler, things might go wrong during instruction scheduling: The non-optimized code of process body P_1 on a typical RISC processor is given in Table 5. The instruction scheduler could now decide to reorder the instructions, e.g. to insert another instruction between a register load and an immediately following register use instruction (e.g. `ldc 1,r0; st r0,a`) which results in the code for P_1 shown in Table 3a. In this case, it may happen that $\textit{critical}_1$ and $\textit{critical}_2$ are both executed, as shown in Table 3b.

ldc	1, r0	Load constant `1` into register `r0`.
st	r0, a	Store `r0` in memory at address `a`.
ld	b, r1	Load content of memory `b`.
cmp	r1, 0	Compare register with constant.
jeq	then_1	Conditional branch to $\texttt{then}_1$.
code of $\textit{else}_1$		
...		

Table 5: Non-optimized code for Process P_1

Even worse, some processors, such as the *DEC Alpha*, are able to reorder the memory accesses to different addresses to some degree. Hence, even the unchanged code could give the wrong result. To avoid this situation, the *DEC Alpha* offers a *memory barrier* instruction, which delays the processor until all pending memory requests are fulfilled. In our example this instructions must follow every memory access, which results in a significant slow-down of the program.